COMMUNICATIONS TO THE EDITOR

Fin Thermal Efficiency during Simultaneous Heat and Mass Transfer

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In the cooling of gas containing a condensable vapor by means of a finned tube heat exchanger there arises the problem of determining the proper fin efficiency for both sensible heat transfer and mass transfer (condensation).

The rate of heat and mass transfer to a bare tube is given by Colburn and Hougen (*1*):

$$dQ = h_g(T_g - t_i)\, dA + K_g \lambda\, (p_v - p_i)\, dA \qquad (1)$$

In the case of a finned tube shown in Figure 1, the temperature of the fin t_i at any point will vary from the tip to the base of the longitudinal fin.

In the case of sensible heat transfer only, the amount of heat transferred to the entire fin is computed by using the temperature of the gas and the temperature of the bare tube surface and a fin efficiency to correct the temperature driving force used to the true average temperature driving force

$$Q_s = h_g\, \Omega_s (T_g - t_b)\, A \qquad (2)$$

where the thermal fin efficiency Ω_s for sensible heat transfer only is given by previous workers (*2, 3*).

An expression similar to Equation (2) for mass transfer can be written

$$Q_c = K_g \lambda\, \Omega_c (p_v - p_i)\, A \qquad (3)$$

where a new efficiency Ω_c is defined in the equation above for diffusion and condensation.

In Equations (2) and (3) above, it has been assumed that the gas-phase composition and temperature in contact with the entire fin are constant and that p_v is greater than p_i. The resistance of the condensate film on the fin has been assumed to be negligible and will be discussed in more detail at the end of this communication.

In the case of simultaneous heat and mass transfer due to condensation, the heat flux at points between the tip of the fin and its base will be different than the heat flux in the case of sensible heat transfer only. As a result the fin efficiencies derived earlier (*2, 3*) on the basis of sensible heat transfer only will not be correct for the simultaneous heat and mass transfer case. The total heat transferred to an increment of area $dA = Sdl$ at any point on the fin is then

$$\frac{dQ}{dl} = h_g(T_g - t_i)\, S + K_g \lambda\, (p_v - p_i) S \qquad (4)$$

The total heat transferred in the entire fin area A can be written as

$$Q_b = h_g\, \Omega_s\, (T_g - t_b)\, A + K_g \lambda\, \Omega_c\, (p_v - p_b)\, A \qquad (5)$$

The fin efficiencies Ω_s and Ω_c are different from each other and are not the same as those given in the literature (*2, 3, 4*).

These efficiencies may be deduced as follows from Equations (4) and (5):

$$\Omega_s = \frac{h_g \int_0^L (T_g - t_i) S\, dl}{h_g\, (T_g - t_b)\, A} \qquad (6)$$

$$\Omega_c = \frac{K_g \lambda \int_0^L (p_v - p_i)\, S\, dl}{K_g \lambda\, (p_v - p_b)\, A} \qquad (7)$$

The overall efficiency Ω_o may be defined as follows:

$$\Omega_o = \frac{h_g \int_0^L (T_g - t_i)\, S\, dl + K_g \lambda \int_0^L (p_v - p_b) S\, dl}{h_g\, (T_g - t_b)\, A + K_g \lambda\, (p_v - p_b)\, A} \qquad (7a)$$

An approximate form of Ω_o was used by Bryan (*5*) for the study of dehumidification of air.

The procedure for calculating fin efficiency will be that given by Kern (*4*). The assumptions given there were used here and will not be restated.

If Q is the flow of heat through the metal fin at any point l

$$Q = -\, k a_x \frac{d t_i}{dl} \qquad (8)$$

$$\frac{dQ}{dl} = -\, k a_x \frac{d^2 t_i}{dl^2} \qquad (9)$$

and substitution of Equation (9) into Equation (4) gives

$$-\frac{d^2 t_i}{dl^2} = \frac{h_g S}{k a_x} (T_g - t_i) + \frac{K_g \lambda S}{k a_x} (p_v - p_i) \qquad (10)$$

Inspection of Equation (10) shows that if p_i is represented by

$$\ln p_i = \overline{A} + B/(t_i + C) \qquad (11)$$

where $\overline{A}$, B, and C are empirical constants, the differential equation will be nonlinear.

NUMERICAL CALCULATIONS

Integration of Equation (10) was carried out on an IBM-1620 computer to obtain a few numerical solutions as follows. The boundary conditions were (see Figure 1)

$$dt/dl = 0 \text{ at } l = L \text{ (that is } Q = 0 \text{ at } l = L)$$
$$t_i = t_b \text{ at } l = 0$$

This presented a trial and error problem in that the two boundary conditions do not apply at the same end. This was solved by estimating the total heat flow at the base and integrating to the tip. A residual heat flow at the tip indicated the estimated heat flow was in error, and an adjustment was made with the two-point modification of

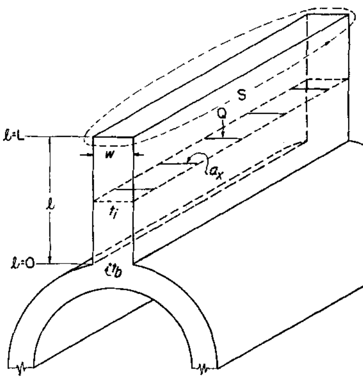

Fig. 1. Description of longitudinal fin.

the Newton-Raphson algorithm used for convergence. Numerical integration was carried out with the Runga-Kutta-Gill algorithm.

Once the proper heat flow and temperature profile were determined, a recomputation was performed in which the sensible heat transfer and the latent heat transfer were separately integrated down the length of the fin. This provided the numerators in Equations (6), (7), and (7*a*) for determining the three efficiencies. The denominators were available directly from original data. Graphical results are given with the following data used for an air-water system at atmospheric pressure:

$t_b = 30°C.$
$T_g = 50°C.$
$h_g = 10$ B.t.u./hr. × sq. ft. × °F.
$$K_g = 1.74 \frac{\text{lb. moles}}{\text{hr.} \times \text{atm.} \times \text{sq. ft.}}$$
$$\lambda = 18{,}800 \frac{\text{B.t.u.}}{\text{lb. mole}}$$
$$k = \left.\begin{matrix} 225 \text{ for copper} \\ 25 \text{ for carbon steel} \end{matrix}\right\} \frac{\text{B.t.u.}}{\text{hr.} \times \text{sq. ft.} \times °\text{F./ft.}}$$
$w = 0.035$ in.
$S/2 = 10$ ft.
$p_v = 0.122$ atm. (water vapor)

Overall efficiency Ω_o is given in Figure 2 for copper and carbon steel, along with those calculated for the case of sensible heat transfer only at the same conditions.

The individual efficiencies Ω_s and Ω_c for the case where condensation is present are shown in Figure 3 for a copper fin. The values of Ω_c are considerably above the Ω_s values because most of the heat flowing through the fin is due to condensation. A comparison of Figure 3 with Figure 2 for copper fins will show that Ω_c has approximately the same value as Ω_o for this set of conditions. In all cases for copper, carbon steel, and stainless steel the values of Ω_o, Ω_c, and Ω_s fell below the efficiency value for the same conditions where only sensible heat transfer was taking place.

CONDENSATE RESISTANCE

The thermal resistance of the condensate film for the cases studied was estimated from the Nusselt equation and found to be of the order of 1% of the gas film heat transfer resistance for the system studied and was, therefore, neglected.

EFFECT OF PARTIAL PRESSURE OF CONDENSABLE VAPOR

Several efficiencies were calculated for various partial pressures of the condensable vapor. These efficiencies are plotted in Figure 4 as a function of the fraction of the total heat transferred that was latent heat. As the partial pressure of the condensable vapor was reduced, the dew point of the gas mixture approached the fin end temperature with a resultant marked decrease in condensation and condensation efficiency Ω_c. At high partial pressures of water vapor the higher rate of condensation results in a steeper temperature gradient along the fin and a decrease in the sensible heat transfer efficiency Ω_s.

APPROXIMATE ANALYTIC SOLUTION

Inasmuch as an analytic solution of Equation (10) cannot be obtained, it was thought that an approximate solution would be useful for cases where the term p_i in Equation (10) can be approximated by the form

$$p_i = \overline{A} + Bt_i \tag{12}$$

$\overline{A}$ and B are empirical constants; then Equation (10) can be integrated as outlined by Kern (*4*) to give

$$Q_b = -(t_b + \alpha/\beta)\, ka_x \sqrt{\beta} \cdot \tanh \sqrt{\beta}\, L \tag{13}$$

$$\Omega_o = Q_b/Q_{max} \tag{14}$$

where Q_{max} is the denominator given in Equation (7*a*).

The constants in Equation (13) are defined as follows:

$$\alpha = -\frac{h_g S T_g}{ka_x} - \frac{K_g S \lambda p_v}{ka_x} + \frac{K_g S \lambda \overline{A}}{ka_x} \tag{15}$$

$$\beta = \frac{h_g S}{ka_x} + \frac{K_g S \lambda B}{ka_x} \tag{16}$$

In deriving Equation (13), it was assumed that the fin temperature t_i was always below the dew point of the

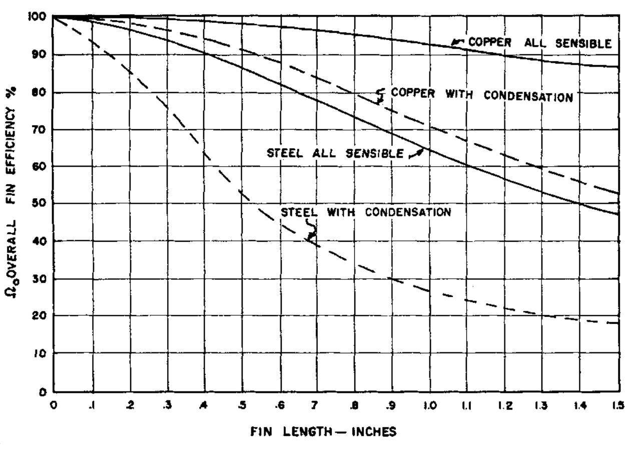

Fig. 2. Reduction of overall fin efficiency due to condensation.

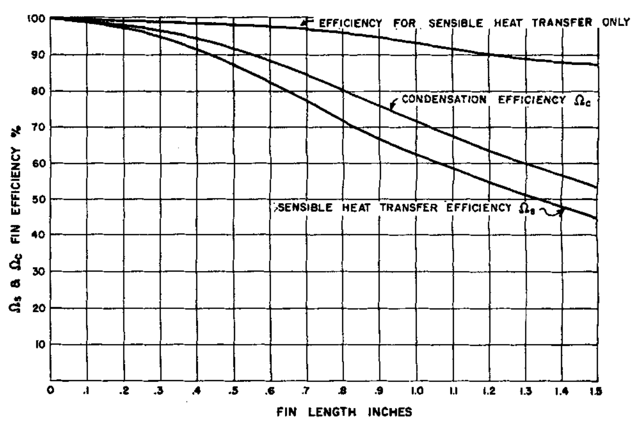

Fig. 3. Comparison of individual fin efficiencies, copper fin.

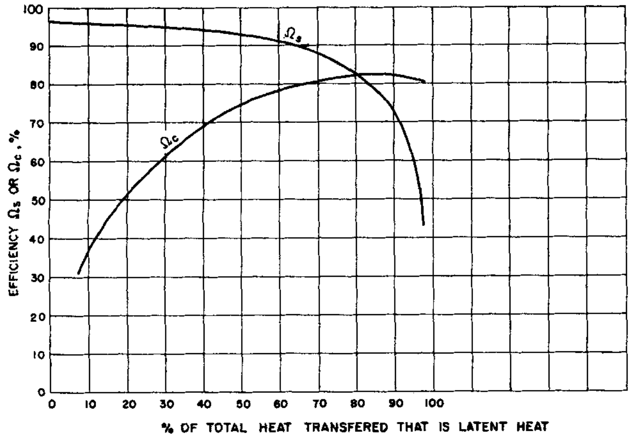

Fig. 4. Variation of fin efficiency with relative rate of condensation, ¾-in. copper fin.

gas such that $p_v > p_i$. When Equation (13) is used, this assumption can be checked by calculating the temperature of the tip or end of the fin by the following expression:

$$t_{\text{end}} = \frac{t_b + \alpha/\beta}{\cosh \sqrt{\beta}\, L} - \alpha/\beta \qquad (17)$$

For highly efficient fins, Equation (13) gives overall efficiencies close to those calculated by integration of Equation (10).

CONCLUSIONS

The fin efficiencies for sensible heat transfer and mass transfer will, in general, not be the same. In the numerical example studied here the sensible heat transfer efficiency approached a maximum value when the fraction of the total heat transferred as latent heat approached zero. In the case where the fin end temperature approached the dew point of the gas the condensation efficiency approached zero, while at the same time the sensible heat transfer efficiency approached its maximum value. Based on this study, the rates of heat and mass transfer to fins must be evaluated by use of individual fin efficiencies to obtain their proper values.

NOTATION

- a_x = cross-sectional area of fin, sq.ft.
- A = area of fin, sq. ft.
- h_g = sensible heat transfer coefficient of gas, B.t.u./hr. sq.ft. °F.
- k = thermal conductivity of metal, B.t.u./hr. sq.ft. °F./ft.
- K_g = mass transfer coefficient, lb. moles/hr. sq.ft. atm.
- l = position along fin perpendicular to base, ft.
- p = partial pressure or vapor pressure condensable vapor, atm.
- Q = heat flow through metal, B.t.u./hr.
- S = perimeter of fin, ft.
- t = temperature, °F.
- T_g = temperature of main body of gas, °F.
- w = fin thickness, in.
- λ = latent heat of condensation, B.t.u./lb. mole
- Ω = thermal efficiency of fin

Subscripts

- b = conditions at the base of the fin
- c = condensation heat effects
- i = condition at gas metal interface
- o = overall conditions
- s = sensible heat effects
- v = vapor

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Note on Dynamics of Liquid-Solid System Expansion and Sedimentation

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Fan, Schmitz, and Miller (*1*) studied the dynamic response of bed height in liquid-solid fluidized bed to a change in fluidizing velocity. Developing previous studies by Slis, Willemse, and Kramers (*2*), these authors derived a linearized analytical model by which they interpreted experimental results. Independently, Massimilla, Volpicelli, and Raso (*3*) studied the properties of pulsing solid-liquid fluidized beds obtained by feeding granular beds with intermittent liquid streams. In this connection, they also investigated a dispersion of dense aggregates of particles in solid-liquid suspensions (*4*).

Fluidization control and pulsing fluidization concern the same problem of unsteady state conditions of a fluidized bed. This is studied by considering two main phenomena, sedimentation of expanded beds and expansion of dense layers of particles.

Solid motion in sedimentation may be regarded as a plug flow, the bed of settled particles being formed pistonlike while a suspension is disappearing. System contraction as a function of the time is evaluted with equations for settling of suspension and material balance of liquid and solid (*3, 5, 6, 7, 8, 9*).

Expansion of a granular bed is a more complex phenomenon. Several models have been suggested to interpret solid flow pattern in expansion as a step up input ($\phi - \phi_{ss}$) occurs (*2, 4*). Piston flow and solid complete mixing models are represented in Figures 1*A*, *B*, and *C*. If particles were rigidly connected to each other, the solid phase